Case Study of Drought Frequency and Risk Analysis in the Upper Green River Basin, Wyoming

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Abstract: The limited length of instrumental streamflow data impacts the true magnitude of natural interdecadal variability of water delivered from the Upper Green River Basin (UGRB). This limited period of instrumental record can be expanded by utilizing proxy records (reconstructed streamflow) derived from tree rings. Recent research has resulted in the development of nine streamflow reconstructions spatially located throughout the UGRB. This paper utilizes four of those nine reconstructed streamflow records and instrumental records to compare and analyze differences between the two streamflow records—human and natural. Three approaches were used for comparison and analysis: (1) Weibull distribution, (2) compound renewal, and (3) drought risk using bivariate probability distribution functions. This analysis has resulted in magnitude-duration-frequency curves for UGRB drought. Such probability curves and stochastic analysis can then be utilized in light of compact agreements and system storage to answer questions such as "How bad is it right now and what can we expect to happen next year?" This case study is intended to show statistical and observed differences between human (short-term) and natural (long-term) streamflow records and specifically target differences in long-term drought characteristics for this drainage basin.
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Introduction

Upper Green River Basin Background

The Upper Green River Basin (UGRB) (Fig. 1) represents a vital water supply for southwestern Wyoming and upper/lower Colorado River compact states. Rapid development in the southwestern United States (e.g., Las Vegas and Phoenix), combined with the recent drought, has greatly stressed the water supply system of the Colorado River. The Colorado River is one of the most physically controlled and regulated rivers in the world. The regulation of the river is known as the law of the river and is comprised of interstate compacts, court decrees, and international treaties. The Colorado River Compact of 1922 divides the Colorado River Basin into two subbasins, with the geographical point of division located approximately at Lee Ferry, Arizona. The lower basin states are Arizona, California, and Nevada, and the upper basin states are Colorado, New Mexico, Utah, and Wyoming. The Colorado River Compact was negotiated by the signatory states and entered into a compact under the Act of Congress of the United States of America and was approved on August 19, 1921. The Colorado River Compact allocated 9.25 billion m^3 (7.5 million acre-ft) annually of water for consumptive use to the Upper and Lower Basins and an annual allocation to Mexico. "Under the hydrologic assumptions of the day, and based on the relatively short period of hydrologic record, the long-term yield of the total watershed was erroneously deemed to be in the range of 19.7 to 21 billion cubic meters (16 to 17 million acre-feet) annually" (Green River Basin Water Plan 2001). Although Wyoming has been annually allocated approximately 1.23 km^3 (1,000,000 acre-ft) of water, the Wyoming State Engineer's Office estimates the probable long-term available water supply for Wyoming from the Green River and its tributaries at 1.03 km^3 (833,000 acre-ft) per year. In terms of land area, the Wyoming portion of the Colorado River Basin is approximately 10%.

Research Background

The limited length of instrumental streamflow data, approximately 100 years, impacts the accurate estimate of natural interdecadal variability of water delivered from the UGRB. This limited period of instrumental record can be expanded by utilizing proxy records (reconstructed streamflow) derived from tree rings. This paper utilizes four reconstructed streamflow records created by Barnett et al. (2010) to analyze drought.

Many differing approaches have been used to characterize and analyze drought events. Answering the question "Are we in a drought?" depends on the working definition of drought, which is assigned by analysis needs. The meaning of the word drought can vary greatly within technical literature (Meko et al. 1995).

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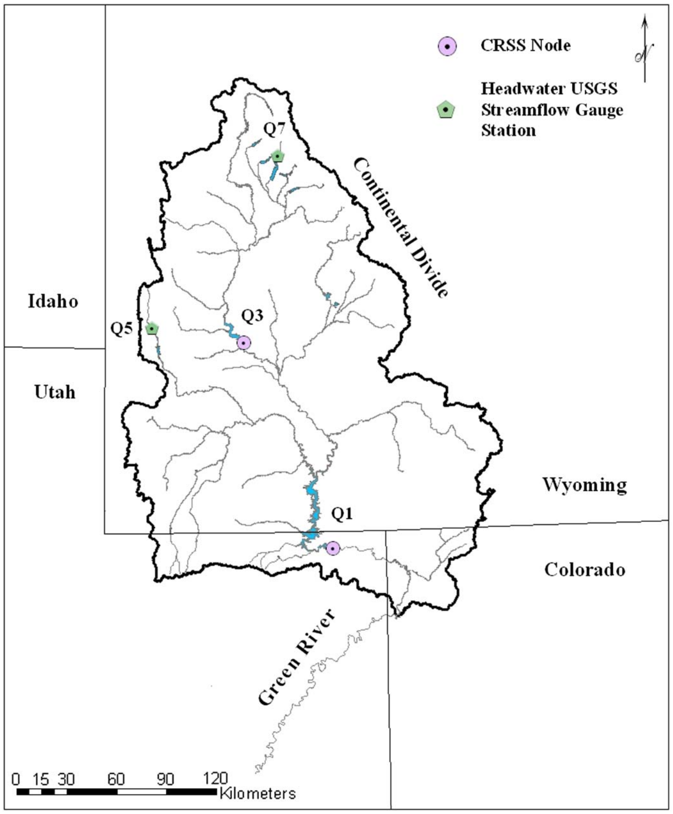

Fig. 1. Upper Green River Basin streamflow gauges [main stem or Colorado River simulation system (CRSS) gauges (Q1, Q3) and headwaters gauges (Q5, Q7)] (map by the authors)

The word drought within society tends to take on an even broader range of definitions and time of reference in which a record length can simply be a person's adult life span to that point and drought severity based on personal hardships. Young (1995) provided two possible definitions of drought based in meteorological terms—limited or no rainfall—or agricultural terms—available soil moisture for evapotranspiration. Loáiciga (2005) defined a drought as an extended period of low streamflows that are not capable of meeting normal water needs. He defined a low streamflow as below the long-term median, and an extended period as one that causes stress to human and environmental uses. An example of this was seen during the summer of 2007 when Atlanta's water storage system was low and water was being released from the system for environmental protection uses. Young (1995) selected a hydrologic measure as a drought indicator that followed the same parameters as Tarboton (1994). This research utilized the drought definition presented by Tarboton (1994) and his referenced authors. As a basis of comparison between the instrumental and reconstructed records, a drought event is defined as a consecutive series of years during which the average annual streamflow is continuously below some specified threshold level, which is typically taken to be the long-term mean (Tarboton 1994). The most recent identified drought for the UGRB generally lasted from 2000–2004, although differing analysis methods may reduce this window or have no drought identified. This drought is used as a defining or threshold event for analysis.

Along with differing definitions, characterizations, and management practices of drought, there are multiple ways that droughts have been probabilistically characterized. Loáiciga (2005) utilized the concept of a compound renewal model for probabilistic analysis

Table 1. USGS Stream Gauges Selected for Drought Analysis

USGS stream gauge information				Basin area		Reconstructed mean		Instrumental mean	
Identification[a]	Identification number	Name	Gauge record	(km^2)	(mi^2)	($m^3 \times 10^6$)	(acre-ft)	($m^3 \times 10^6$)	(acre-ft)
Q1	9234500	Green River near Greendale, UT	1906–2004[b]	50,116	19,350	2,351	1,906,008	2,456	1,991,010
Q3	9211200	Green River below Fontenelle Reservoir, WY	1906–2004[b]	11,085	4,280	1,573	1,275,290	1,639	1,328,771
Q5	9223000	Hams Fork near Frontier, WY[c]	1953–2004	332	128	84	68,457	85	68,615
Q7	9196500	Pine Creek above Freemont Lake, WY	1955–2004	197	76	153	123,845	155	125,953
				Q1 Naturalized		1906–1995	1971–1995	1971–2000	1971–2004
				Long-term mean		2,481	2,506	2,565	2,428
				($m^3 \times 10^6$/acre-ft)		2,010,990	2,031,922	2,079,686	1,968,205

[a] Gauge identification shown on map in Fig. 1.

[b] Record is naturalized flow (U.S.B.R.).

[c] Full gauge name: Hams Fork below Pole Creek, near Frontier, WY.

of drought events. This process is a generalization of the Poisson approach to prediction, in which the probability distribution function (pdf) of drought duration and the interarrival time are identified and tested directly from data. The Poisson process describes the occurrence of random events during an interval time. The number of events during that interval becomes a discrete random variable that can be described by a Poisson distribution, and the interval length between events can be modeled by an exponential distribution (Montgomery et al. 2004). The approach utilized by Salas et al. (2005) was an extension of the work presented by Yevjevich (1967) in which joint probability distributions were used to characterize associated probabilities of drought duration and magnitude.

Increasing the length of streamflow records can provide more accurate frequency and risk assessment of drought events. Instrumental gauge records, naturalized or unimpaired, do not always provide a data population that is long enough to truly understand long-term natural variability within a river system. To counteract this problem, some researchers have used stochastic models to generate streamflow data (Salas et al. 2005).

Rather than use stochastic models, this research utilized reconstructed records from tree ring chronologies to extend the instrumental record. The reconstructed records encompass a period from 1615–2004. This period is believed to be of sufficient length to provide a large enough data population to counteract problems associated with a limited instrumental record.

The following presents this approach by first summarizing the data and methods used for comparison and analysis and then comparing differences between results using the instrumental, human record and reconstructed, natural record for analysis and conclusions. This case study is intended to show statistical and observed differences between human, short-term and natural, long-term streamflow records and specifically target differences in long-term drought characteristics for this drainage basin.

Data

Four streamflow reconstructions from Barnett et al. (2010) were used for drought analysis in the UGRB. The four stream gauges were denoted as Q1, Q3, Q5, and Q7 (Table 1, Fig. 1). Stations Q1 and Q3 were located on the main stem of the Green River, whereas gauge stations Q5 and Q7 were located on lower order tributaries further up in the watershed. Gauge records for Q1 and Q3 were naturalized records, whereas records for Q5 and Q7 were unimpaired. Naturalized records have been adjusted to account for human influences, and unimpaired records were considered to have limited human impact and have not been adjusted. Both types of records were used in this study, and both are comparable because they both depict streamflow without human influences. The USGS gauge identification number and associated length of instrumental record is reported with the reconstructed and instrumental means (Table 1). The instrumental records for comparison were obtained from the U. S. Bureau of Reclamation (Prairie 2004). For each gauge station, the reconstructed mean for the period of 1615–2004 is below that of the instrumental mean. The bottom of Table 1 also shows means for differing time periods of the Q1 instrumental record. Dry periods in the late 21st and early 20th centuries are responsible for the mean of the 1971–2004 record to be less than that of the 1906–1995 record. Barnett et al. (2010) utilized tree ring chronologies to create reconstructed records that were integrated with gauge records to produce the final water year records used in this paper (Fig. 2). The final reconstructed records were rescaled to force the standard deviation in the reconstruction to be equal to that of the stream gauge record (Timilsena et al. 2007). This adjustment was performed so that the stream gauge record could be joined with the reconstructed time series. Fig. 2 represents the reconstructed water year—the sum of all flow through the point in the river over one year.

Methodology

Anomalies in a gauge record are identified by subtracting a threshold value from an individual reconstructed water year's value (x_t). Fig. 3 is a bar chart of anomalies from gauge station Q1's reconstructed record. The threshold used in this part of the study is the long-term mean (μ) of the entire reconstructed record (1615–2004). When an individual water year value is greater than the threshold, the anomaly will have a positive value, or is considered a wet or surplus year (s_t). When the water year value is below the defined threshold, a negative anomaly is seen and is considered a dry or deficit year (d_t). The upper bars in Fig. 3 represent wet years or positive anomalies, whereas the lower bars represent dry years. Each bar represents 1 year.

Drought events can be identified by multiple methods, and are defined in the next section. The duration (L_i) of a drought event is

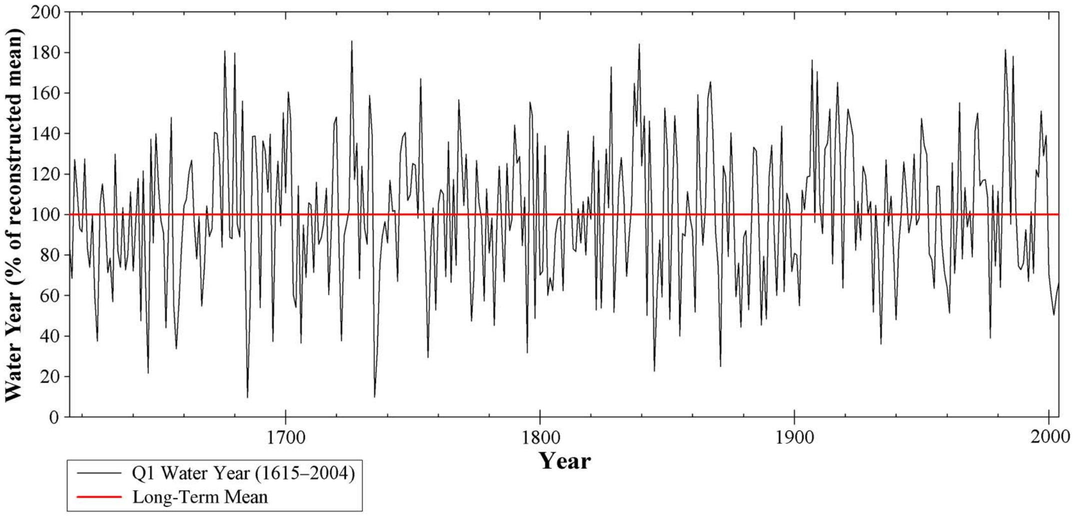

Fig. 2. Reconstructed water year streamflow (1615–2004) expressed as percentage of long-term mean for Green River near Greendale, Utah (Q1)

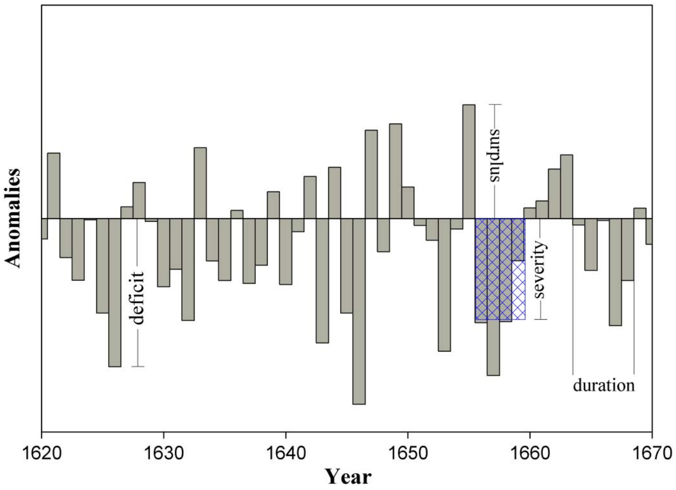

Fig. 3. Reconstructed water year streamflow anomalies (1620–1670) for Green River near Greendale, Utah (Q1)

the sum of consecutive years with values below the mean ($x_t < \mu$). The duration represented in Fig. 3 is of the 1664–1668 event that lasted 5 years ($L_i = 5$). The beginning year of a drought is the year when $x_t < \mu$, and the end year is the last consecutive year below the threshold. The magnitude (M_i) of a drought is the sum of all deficits included in the duration or cumulative deficit. The severity (S_i) of a drought is obtained by dividing magnitude by duration (M_i/L_i) or the average deficit of an event. The cross hatched box represents the severity of the 1,656–1,659 event, $S_i = (d_{1,656} + d_{1,657} + d_{1,658} + d_{1,659})/L_i$ where $L_i = 4$.

Drought Identification Methods for Comparison

Drought events were identified by five drought identification methods similar to those used by Timilsena et al. (2007). They are denoted as drought identification method (a)–(e). Method (a) only considers droughts with duration of 2 years or more. For example, the deficit at 1648 would not be considered a drought event because of the positive anomalies on each side (Fig. 3). As stated by Timilsena et al. (2007), this method does not take long periods of drought into account because a single x_t value equal to or above the threshold (creating a surplus) causes the duration to end. Longer dry periods with multiyear averages below the threshold are analyzed by taking moving averages of the entire record, also called filtering. Moving averages of 3, 5, 7, and 10 years are offset at the beginning of the reconstructed record to include the end year of 2004. These methods are denoted (b), (c), (d), and (e), respectively. Drought identification methods (a)–(e) afford a quantitative comparison among identified droughts in a single record and common events between records. These methods can also be called filtering the record. The return periods in Table 2 are only shown as a comparison between the two. Drought identification methods are not used for comparisons between methods 1 and 3.

Methods for Return Periods and Renewal Times

Three methods—the Weibull, the compound renewal, and the drought risk methods—were used to calculate occurrence probabilities for events from the entire Q1 reconstructed record (1615–2004). Method 1 calculates return periods from a single variable. Method 2 uses two variables; however, like Method 1, return periods cannot be greater than the number of events listed, and events outside of the record cannot be investigated.

Return periods and renewal times for stations Q3, Q5, and Q7 are reported in Table 2; however, they are not used for comparison to method 3 in this case study so that the amount of comparison data does not become overwhelming. Station Q1 is defined as the basin outlet and receives flow from stations Q3, Q5, and Q7. It is believed to incorporate all basin characteristics.

Weibull (Method 1)

The Weibull formula is a simple, rapid, and commonly used method to calculate exceedance probabilities and their associated

Table 2. Drought Rankings Sorted for Magnitude

Gauge station	Drought identification method	Event rank					Average duration (years)	Rank of recent drought	Method 1 return period	Method 2 renewal time
		1st	2nd	3rd	4th	5th				
Q1	a	1735–1740	1877–1883	2000–2004	1844–1848	1656–1659	3	# 3 (2000–2004)	130	132
	b	1630–1648	1704–1719	1653–1661	1878–1884	1736–1742	4	# 8 (2001–2004)	49	52
	c	1623–1650	1932–1944	1653–1662	1706–1718	1735–1742	6	# 17 (2002–2004)	23	23
	d	1624–1652	1879–1895	1931–1946	1656–1664	1706–1719	7	# 19 (2003–2004)	21	21
	e	1624–1655	1877–1906	1657–1673	1933–1949	1801–1818	9	# 18 (2003–2004)	22	22
Q3	a	1877–1883	1844–1848	1803–1809	2000–2004	1735–1740	3	# 4 (2000–2004)	98	65
	b	1800–1810	1878–1884	1704–1715	1844–1849	1653–1660	4	# 8 (2001–2004)	49	49
	c	1878–1894	1622–1641	1931–1944	1801–1811	1845–1852	6	# 16 (2002–2004)	24	25
	d	1879–1906	1624–1651	1931–1946	1803–1813	1706–1719	7	# 16 (2003–2004)	24	25
	e	1624–1673	1877–1908	1933–1949	1801–1816	1708–1719	10	# 15 (2003–2004)	23	23
Q5	a	1703–1711	1803–1809	2000–2004	1844–1848	1931–1936	3	# 3 (2000–2004)	130	148
	b	1704–1713	1630–1639	1802–1810	1989–1996	1896–1906	4	# 8 (2001–2004)	49	52
	c	1619–1640	1704–1715	1804–1811	1896–1906	1933–1941	5	# 12 (2001–2004)	33	34
	d	1879–1907	1621–1641	1704–1718	1803–1813	1934–1942	6	# 14 (2002–2004)	28	29
	e	1624–1655	1878–1908	1706–1718	1804–1815	1993–2004	8	# 5 (1993–2004)	78	90
Q7	a	1703–1711	1803–1809	1629–1632	1931–1936	1844–1848	3	# 20 (2000–2004)	20	52
	b	1630–1639	1704–1713	1802–1810	1932–1938	1897–1904	4	# 19 (2001–2004)	21	23
	c	1619–1641	1887–1906	1704–1713	1804–1811	1933–1941	6	# 19 (2002–2004)	21	21
	d	1621–1643	1880–1907	1803–1813	1706–1715	1934–1941	6	# 20 (2004)	20	20
	e	1624–1655	1878–1908	1802–1815	1706–1718	1935–1943	7	No drought	No drought	No drought

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return period. This formula was used for comparison in this study because of its ease of calculation and the rapid generation of a return period (100-year drought). When drought events are sorted according to magnitude, the probability (ρ) and return period (R) can be determined by a Weibull distribution [Eqs. (1) and (2)]. Because the events are sorted from largest to smallest with respect to their absolute value, the calculated probability becomes an exceedance probability [$\rho(X \geq x_t)$]

$$\rho = \frac{m}{(N+1)} \tag{1}$$

$$R = \frac{1}{\rho} \tag{2}$$

$$N = \Sigma(\text{all recorded events, 1615–2004}) = 390 \tag{3}$$

The parameter m = rank of an event, with the largest drought in magnitude or severity ranked 1; and R is in years. The Weibull solutions for event probability, $m/(N+1)$, equal the average exceedance probability of the ranked observations x_t, and hence are probability-unbiased solutions (Stedinger et al. 1993). Return periods were identified for each drought identification method and are reported as method 1(a)–1(e).

Compound Renewal (Method 2)

The second analysis technique uses a compound renewal approach (Timilsena et al. 2007; Loáiciga 2005). This method is a generalization of the Poisson process, in which the pdf for duration of and interval time between droughts are identified and sampled directly from data. Method 2 considers an event's magnitude and its duration. This is accomplished by identifying drought events that are equal to or greater than a defined event. All identified events must equal or exceed the defining event's threshold duration (θ) and magnitude (M_i) simultaneously. Any event contained within the record can be selected to provide threshold values, depending on analysis needs. The defining event for this research is the most recent drought (approximately 2000–2004), L_{recent} ($\theta = 5$ years; i.e., length of recent drought; and $M_i = 3{,}653{,}818$ acre-ft or 4.51 billion m^3; i.e., the cumulative deficit over 5 years). As with method 1, the events are classified as method 2(a)–2(e). Loáiciga (2005) defined $\bar{R}$ as

$$\bar{R} = \theta + \frac{1}{a_1} + \frac{1}{a_2} \tag{4}$$

$$a_1 = \frac{1}{(\bar{L} - \theta)} \tag{5}$$

$$a_2 = \frac{1}{\bar{T}} \tag{6}$$

The parameters a_1 and a_2 = average duration ($\bar{L}$) and average interval time ($\bar{T}$) between selected events (i.e., those events $\geq \theta$ and $\geq M_i$). Eq. (4) assumes that $a_1 \neq a_2$. The time before the first selected event and the time after the last selected event are not considered in the average interval time.

Drought Risk (Method 3)

The third analysis technique characterizes droughts by defining two cases of drought events to analyze event probability and risk (Salas et al. 2005). Case 1 events [Eq. (7)] are characterized by droughts with accumulated deficit (D) greater than a specified deficit (D_0) and duration (L) equal to an analyzed event's duration or threshold duration (l_0). For method 3, accumulated deficit is the same as an event's magnitude, and the specified deficit is the same as the threshold magnitude. If desired, case 1 droughts can be analyzed for duration only, by setting the specified deficit to zero ($D_0 = 0$). Case 2 events [Eq. (8)] utilize the same analytical procedures as case 1 by replacing deficit (magnitude) with intensity (severity, i.e., $I = L/D$). The occurrence probability of each case is described by the bivariate probability distribution functions shown in Eqs. (7) and (8). Each expression follows a gamma distribution characterized by their individual shape (r) and scale (β) factors. The shape and scale factors used in the equations were obtained by fitting all identified droughts for each method 1 scenario to a gamma distribution. Each expression is also defined by a prescribed length (l_0) and deficit (D_0) or intensity (I_0), in which intensity is the same as severity, as defined previously. Transition probabilities (p_{xx}) can be calculated for each water year record with Eqs. (9) and (10) (Jackson 1975; Fernández and Salas 1999). It is assumed that the sequence of surpluses (denoted by 1) and deficits (denoted by 0) follow a Markov chain. The number of transitions (N) from fail (f) to safe (s) or 0 to 1 is the number of times that the water year record transitions from a drought event to a surplus event. This process is also used for transitions from safe to fail, fail to fail, and safe to safe. The return period (T) of any event described by Salas et al. (2005) can be evaluated with Eq. (11)

$$P(D > D_0, L = l_0) = \int_{D_0}^{\infty} \frac{1}{\beta\Gamma(l_0 r)} \left(\frac{z}{\beta}\right)^{l_0 r - 1} e^{-z/\beta} p_{01}(1 - p_{01})^{l_0 - 1} dz \tag{7}$$

$$P(I > I_0, L = l_0) = \int_{I_0}^{\infty} \frac{l}{\beta\Gamma(l_0 r)} \left(\frac{l_0 z}{\beta}\right)^{l_0 r - 1} e^{-l_0 z/\beta} p_{01}(1 - p_{01})^{l_0 - 1} dz \tag{8}$$

$$p_{fs} = p_{01} = \frac{N_{fs}}{N_{ff} + N_{fs}} \tag{9}$$

$$p_{sf} = p_{10} = \frac{N_{sf}}{N_{ss} + N_{sf}} \tag{10}$$

$$T = \frac{P_{01} + P_{10}}{P_{01} P_{10}} \frac{1}{P(E)} \tag{11}$$

Results

Drought identification methods (a)–(e) were analyzed and ranked from largest to smallest magnitude or severity, with the largest or most negative magnitude/severity ranked first. The method 1 return periods and method 2 renewal times were also calculated for the reference drought event for each case. This process was repeated for each of the selected gauge stations.

Drought Identification Methods (a)–(e) and Methods 1 and 2

Rankings, sorted for magnitude, for methods 1 (Weibull) and 2 (compound renewal) and drought identification methods (a)–(e) are summarized in Table 2. The average duration column in Table 2 represents the average duration of all identified events within the entire reconstructed record. The rank of recent drought column lists the rank of the reference drought and the consecutive years of negative anomalies for that event. The two right-hand columns

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Table 3. Drought Rankings Sorted for Severity

Gauge station	Drought identification method	Event rank					Rank of recent drought	Number of identified droughts
		1st	2nd	3rd	4th	5th		
Q1	a	1645–1646	1684–1686	1656–1659	1703–1704	2000–2004	# 5 (2000–2004)	51
	b	1685–1687	2001–2004	1736–1741	1871–1873	1887–1890	# 2 (2001–2004)	47
	c	2002–2004	1878–1884	1735–1742	1653–1662	1667–1671	# 1 (2002–2004)	35
	d	1656–1664	1803–1811	1846–1852	1624–1652	1735–1745	# 7 (2003–2004)	29
	e	1736–1745	1624–1655	1657–1673	1960–1970	1877–1906	# 19 (2003–2004)	23
Q3	a	1703–1704	1625–1626	1645–1646	1684–1686	1844–1848	# 7 (2000–2004)	50
	b	2001–2001	1878–1884	1685–1687	1844–1849	1736–1740	# 1 (2001–2004)	52
	c	2002–2004	1845–1852	1645–1661	1801–1811	1991–1996	# 1 (2002–2004)	37
	d	1846–1852	1656–1663	1803–1813	1992–1996	1879–1906	# 6 (2003–2004)	31
	e	1877–1908	1801–1816	1933–1949	1708–1719	1624–1673	# 15 (2003–2004)	21
Q5	a	1919–1920	1684–1686	1901–1902	1625–1626	1886–1887	# 8 (2000–2004)	42
	b	2001–2004	1846–1849	1685–1687	1704–1713	1802–1810	# 1 (2001–2004)	53
	c	1804–1811	2001–2004	1845–1852	1989–1996	1756–1761	# 2 (2001–2004)	40
	d	1846–1853	1803–1813	1621–1641	2002–2004	1934–1942	# 4 (2002–2004)	32
	e	1804–1815	1706–1718	1624–1655	1936–1943	1878–1908	# 6 (1993–2004)	26
Q7	a	1919–1920	1625–1626	1684–1686	1629–1632	1721–1722	# 37 (2000–2004)	45
	b	1846–1849	1685–1687	1630–1639	1704–1713	1802–1810	# 17 (2001–2004)	48
	c	1804–1811	1704–1713	1933–1941	1845–1852	1619–1641	# 12 (2002–2004)	36
	d	1706–1715	1934–1941	1803–1813	1846–1853	1621–1643	# 16 (2004)	35
	e	1624–1655	1706–1718	1802–1815	1935–1943	1878–1908	No drought	29

of Table 2 list the return period (method 1) and the renewal time (method 2) of the reference drought for each drought identification method.

Rankings, sorted for severity, for the Weibull, compound renewal, and drought identification methods (a)–(e) are summarized in Table 3. The last column of Table 3 lists the number of droughts that were identified from the full reconstructed record as each drought identification method was considered.

As shown, the magnitude of the most recent drought (approximately 2000–2004) is a notable event, but it is not the greatest event of record. Although there are a few exceptions, the ranking of the recent drought and return periods and renewal times for methods 1 and 2 diminish as longer, end year, moving averages are considered (Table 2). This shows that the years leading up to the reference event were, on average, in surplus. The aforementioned trend does not exist when severity is considered (Table 3). Results show that the recent event does not exhibit the greatest magnitude for drought events; however, with the exception of gauge Q7, its severity is the greatest when examining a 3–5 end year average.

Method 3

Eq. (7) was solved in two manners. The first, Fig. 4, illustrates the solutions for method 3, case 1 when only duration is considered ($D_0 = 0$). When D_0 is set to zero, the first portion, or the pdf portion $\{[1/\beta\Gamma(l_0 r)](z/\beta)^{l_0 r-1}e^{-z/\beta}\}$, of Eq. (7) is integrated from zero to infinity, and by definition is equal to one. The resulting event probability is calculated from the second portion of the equation, or the transitional probabilities portion $[p_{01}(1-p_{01})^{l_0-1}]$. The second portion of Eq. (7) is a constant and can be evaluated outside of the integral. The solutions for each gauge station are plotted together with Q5 and Q7 overlapping one another. These return periods can be viewed as minimum or threshold return periods. For the reference drought, the minimum return period associated with an event having a 5-year duration is 107 years. Fig. 5 shows a graphical solution for method 3, case 1 solutions when an event's magnitude is also considered. Fig. 5 can be used to ascertain the associated cumulative deficit or magnitude of an event of interest. The graph is entered from the x-axis by selecting a duration and then following that vertically to a desired return period. Once the desired return period is intersected, it is moved horizontally to the y-axis where the magnitude can be read, expressed as both millions of acre-ft and millions of cubic meters. Method 3, case 2 events were calculated; however, they have a poor fit, as shown in Table 4.

Comparisons

There are notable differences in the threshold return periods calculated by methods 1 and 3, shown in Table 5. For example, when station Q1 is sorted for magnitude, the 1735–1740 event is ranked first and has the highest method 1 return period of 391 years and

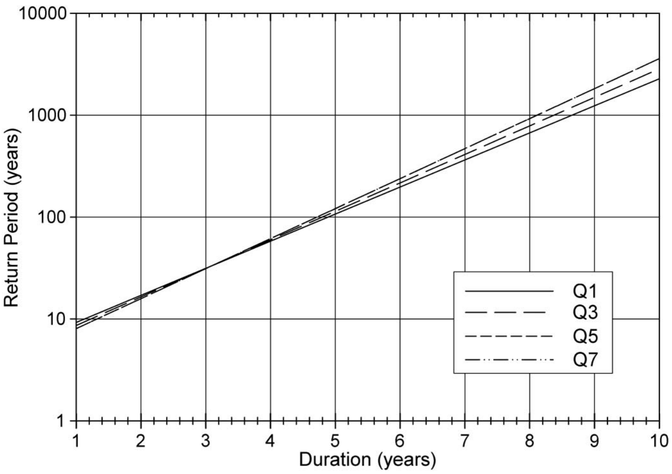

Fig. 4. Drought duration (years) and return period (years) for Q1, Q3, Q5, and Q7 for case 3, method 1

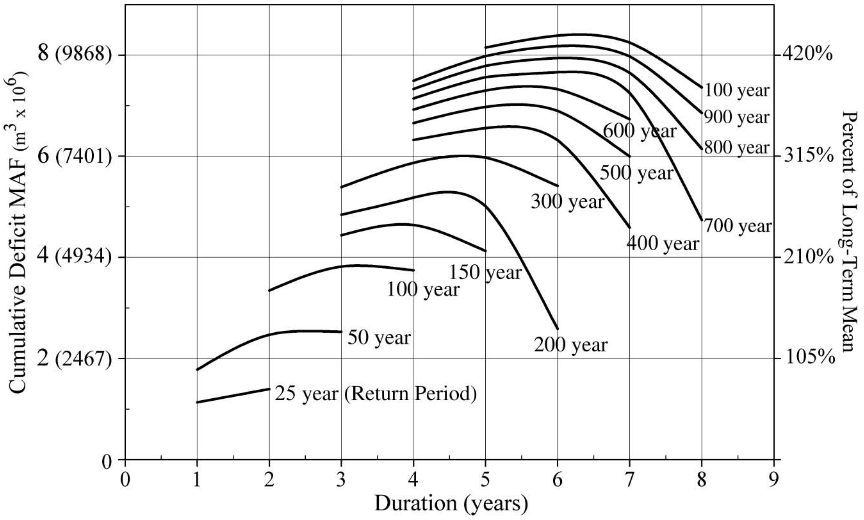

Fig. 5. Magnitude [million acre-feet (MAF) or million cubic meters (million m^3)], duration (years), and frequency (years) curves for Q1 for case 3, method 1

a duration of 6 years. If the duration of the 1735–1740 event is considered as in method 3, the minimum return period associated with that event decreases to 197 years. When the event's duration is considered in conjunction with its deficit, the return period is raised to 225 years. Because method 1 does not consider duration as a factor, the 1877–1883 event is ranked second, simply because its magnitude is approximately 0.43 billion m^3 (0.35 million acre-ft) less. This ranking results in a method 1 return period of 196 years. For the 1877–1883 event, when duration is considered, the minimum return period increases to 363 years.

Even greater differences occur for threshold return periods between methods 1 and 3 when the rankings are according to the event's associated severity (method 3, case 2). Although these results are reported (Table 6), the gamma distribution fit for the reconstructed record severity (intensity) is not very good. This is because of high Anderson-Darling statistics and a p-value that is less than the significance level (0.05).

Table 7 compares the return periods for events with durations from 1–10 years for the reconstructed versus instrumental records, for Q1, calculated using Eq. (7). The return periods for common events, events with short durations, are relatively equal between the two data sets until a duration of 5 years. At that point, the reconstructed return periods do not grow as quickly as those of the instrumental record. The difference between calculated return periods is because of a difference in transitional probabilities between records. The transitional probability going from a drought to a surplus for the instrumental record is $P_{01,\text{instr}} = 0.469$, and for the reconstructed record $P_{01,\text{reconstr}} = 0.458$ (Table 5).

Table 4. Gamma Distribution Information and Transition Probabilities for Method 3 at Q1

Magnitude		Shape (α)	Scale (β)	p_{01}	p_{10}	AD	P-value
Reconstructed record (magnitude)	Dry	1.352	807585	0.4577	0.4894	0.232	>0.250
	Wet	1.130	977487	0.4577	0.4894	0.336	>0.250
Instrumental record (magnitude)	Dry	1.170	905134	0.4694	0.4898	0.471	>0.250
	Wet	0.7262	1457664	0.4694	0.4898	0.442	>0.250
Reconstructed record (intensity)	Dry	2.477	201937	0.4577	0.4894	0.851	0.033
	Wet	2.122	236899	0.4577	0.4894	1.068	0.01
Instrumental record (intensity)	Dry	2.563	191164	0.4694	0.4898	0.276	>0.250
	Wet	1.079	430291	0.4694	0.4898	0.493	0.243

Note: AD = Anderson-Darling test for gamma distribution; P-value = P-value for Anderson-Darling statistic; 95% confidence interval (CI) for all distributions.

Table 5. Threshold Return Period Comparison between Methods 1 and 3, Sorted for Magnitude

Gauge station	Rank (sorted for magnitude)	Date	Duration	Magnitude ($m^3 \times 10^6$)	Method 1 return period (years)	Method 3 return period (duration only)	Method 3 return period (years)
Q1	1	1735–1740	6	4,970	391	197	225
	2	1877–1883	7	4,536	196	363	374
	3	2000–2004	5	4,507	130	107	133
	4	1844–1848	5	4,065	98	107	124
	5	1656–1659	4	4,035	78	58	84
	6	1803–1809	7	3,752	65	363	367

Table 6. Return Period Comparison, Severity

Gauge station	Rank (sorted for severity)	Date	Duration	Severity ($m^3 \times 10^6$)	Method 1 return period (years)	Method 3 return period (duration only)	Method 3 (case 2) return period (years)
Q1	1	1645–1646	2	1,394	391	17	1,314
	2	1684–1686	3	1,226	196	31	2,436
	3	1656–1659	4	1,009	130	58	1,567
	4	1703–1704	2	1,007	98	17	185
	5	2000–2004	5	900	78	107	1,648
	6	1755–1757	3	886	65	31	259
	19	1877–1883	7	648	21	363	938

Table 7. Return Period Comparison, for Duration Only, at Q1

Reconstructed record		Instrumental record	
Duration (years)	Return period (years)	Duration (years)	Return period (years)
1	9	1	9
2	17	2	17
3	31	3	32
4	58	4	59
5	107	5	112
6	197	6	211
7	363	7	398
8	670	8	751
9	1,235	9	1,415
10	2,278	10	2,666

Example

The following is an example depicting the potential use of the information presented herein. The example uses the drought risk procedure (method 3) to calculate return periods and probabilities. Assume the Upper Green River Basin has a total storage for release to the Colorado River system of approximately 3 years mean flow, or 7.05 billion m^3 (5,718,000 acre-ft). Also assume the system is currently in a drought that has lasted 2 years, the system storage is now at approximately 80%, and the system has been experiencing submean flows for the last 2 years with a magnitude of 1.23 billion m^3 (1,000,000 acre-ft). The current assumed situation, neglecting natural losses, is a drought event with a 2-year duration that has a minimum probability of 0.2482 and a return period of 17 years with $D_0 = 0$ and $l_0 = 2$ (using data from Table 4 for the reconstructed record). The event probability is $P(D > D_0, L = l_0)$, where $D_0 = 1{,}000{,}000$ acre-ft or 1.23 billion m^3 and $l_0 = 2$. The gamma shape factor $r = 1.35$, the scale factor $\beta = 807585$, $P_{01} = 0.4577$, and $P_{10} = 0.4894$ (Table 4, reconstructed record). From Eq. (7),

$$P(E) = \int_{1{,}000{,}000}^{\infty} \{1/[807{,}585 * \Gamma(2 * 1.35)]\}(z/807{,}585)^{2*1.35-1} \times e^{-z/807{,}585} 0.4577(1-0.4577)^{2-1} dz = 0.2038$$

and from Eq. (11),

$$T = \frac{0.4577 + 0.4894}{0.4577 * 0.4894} \frac{1}{0.2038} = 21 \text{ years}$$

What is going to happen next year? The minimum probability to go into a third year of drought is $P(D > 0, L = 3) = 0.1346$, and $T = 31$ years. The probability of reducing the system storage to 70%, neglecting natural losses, in that year is the loss of an additional 7.05 billion m^3 (571,800 acre-ft) for $D_0 = 1{,}715{,}400$ acre-ft or 2.11 billion m^3 and $l_0 = 3$; $P(E) = 0.1132$ and $T_r = 37$ years. The probability of coming out of the drought is $1 - P(D > 0, L = 2)$ or 0.7518. The probability of restoring the system storage from 80 to 90% in 1 year requires a wet year with a surplus of 7.05 billion m^3 (571,800 acre-ft). To calculate this probability, Eq. (7) must be redefined from deficit to surplus like in Eq. (12), in which transitional probabilities are also redefined from from surplus to deficit

$$P(S > S_0, L = l_0) = \int_{s_0}^{\infty} \frac{1}{\beta\Gamma(l_0 r)} \left(\frac{z}{\beta}\right)^{l_0 r-1} e^{-z/\beta} p_{10}(1-p_{10})^{l_0-1} dz \quad (12)$$

This can be accomplished by replacing the deficit term in Eq. (7) with a surplus and replacing the transitional probability of going from a drought to a surplus (ρ_{01}) with the probability of transitioning from a surplus to a drought (ρ_{10}).

$$P(S > 571{,}800, L = 1) = P(S) = \int_{571800}^{\infty} \frac{1}{977487 * \Gamma(1 * 1.13)} \left(\frac{z}{977487}\right)^{1*1.13-1} \times e^{-z/9977487} 0.4894(1-0.4894)^{1-1} dz = 0.3025$$

Assuming that the probabilities of exiting a drought and having a sufficient surplus occur in the next (third) year is independent, the probability of restoring the system to full capacity in the third year is equal to $[1 - P(D > 0, L = 2)] * [P(S > 571{,}800, L = 1)] = 0.7518 * 0.3025 = 0.2274$.

Discussion and Conclusions

The use of dendrohydrology to create reconstructed streamflow records can improve stochastic estimates of streamflow variations by extending the available period of record. The reconstructed period of record contains drought events that surpass any events that would be part of the instrumental record in terms of magnitude and duration. For the Upper Green River, the long-term means for instrumental and reconstructed records are of notable difference (Table 1). This is caused by the wet period in the early twentieth century, exemplified in the instrumental record, and the long dry period exemplified in the early seventeenth century.

Methods 1 and 2 are directly dependent on the reconstructed record for the calculation of return periods/renewal times. Method 1 return periods are limited to bin sizes that are dependent on the record length, as seen in Eq. (1). The difference between return periods becomes more pronounced at the less probable values. The largest possible return period for and event in the reconstructed record, utilizing method 1 (Weibull), is 391 years,

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and is only dependent on one variable, duration or magnitude. This singular dependence causes method 1 to be a poor choice when calculating return periods. Method 2 is similarly limited to renewal times based on the number of observed events and the time between. Method 2 is also incapable of looking at events larger than those contained in the record used. Method 3 is based on the same record; however, it allows for the interpolation of greater return periods/renewal times than methods 1 and 2. A general visual inspection of the reconstructed record shows that the twentieth century, the period of the instrumental record, was dominated by higher streamflows than earlier periods. If only this time period were to be considered, the descriptive statistics such as median and mean would be artificially elevated to the same statistics of the entire reconstructed record. The results also show evidence of this in the comparison of return periods between the instrumental and reconstructed records, with the reconstructed return periods being lower. Comparisons between the two records are validated because the reconstructed records were integrated with the gauge records. When the reconstructed records were integrated with the instrumental gauge records, the overlapping periods of records (for Q1 this was 1906–2004) became identical, and the preceding portion of the reconstructed record was scaled to fit the standard deviation of the instrumental record.

Finally, it should be noted that all estimates calculated by method 3 are point estimates. The standard errors of these estimates are not calculated, and the reported results should not be taken as absolute, but with the understanding that there is uncertainty associated with the estimates.

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